GARCH Models

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This tutorial focuses on volatility modelling of financial returns. The objective of the tutorial is to present the main model specifications, to derive their probabilistic properties and to analyse the relevant inference methods. The key model of interest is the generalised autoregressive conditionally heteroscedastic (GARCH) process introduced by Engle. The probabilistic and statistical properties of this time series model will be studied and confronted with the stylised facts of the financial series. We will also focus on the multivariate framework of volatility modelling, called Multivariate GARCH: both probabilistic and statistical properties will be considered.

Tutorial 1: GARCH processes

Risk modelling and/or option pricing often require the modelling of the variance of the underlying asset. Hence GARCH-like processes are often used in the financial industry. We consider the univariate specification of the conditional volatility GARCH(p,q) model and motivate its use through financial applications. We also focus on the conditions to derive its stationarity property and the estimation method based on the Gaussian Quasi Maximum Likelihood method.

1. GARCH modelling for financial series.
2. Stationarity of GARCH processes.
3. Estimation of GARCH processes: QML estimator.

Tutorial 2: GARCH and stochastic volatility model

Starting from the pros and cons of the GARCH process, we introduce the univariate stochastic volatility model together with the inference method. Then we consider the hidden-Markov - or Markov switching (MS) - model, which is a significant parameterization among the family of dynamic models with hidden variables. This modelling is intuitive and can easily be interpreted for financial time series.

1. General stochastic volatility model.
2. Estimation.
3. Markov switching models.

Tutorial 3: Multivariate GARCH models

The multivariate setting is nowadays required when performing risk analysis, where the objective is to provide a prediction of the variance/covariance matrix of a portfolio of diversified assets. To do so, we study the framework of multivariate GARCH model

(MGARCH) and consider several variance covariance and correlation matrix processes. We also consider the probabilistic properties of such dynamics and the asymptotic properties of the Constant Conditional Correlation model will be derived.

1. MGARCH processes: VEC-GARCH, BEKK, DCC.
2. Stationarity of VEC-GARCH, BEKK and CCC models.
3. Estimation of CCC model: analysis of the QML estimator.

References

- Brockwell and Davis, (1991) *Time Series: Theory and Methods.* Springer.
- Francq, C and Zakoïan, J.M. (2010). *GARCH Models : Structure, Statistical Inference and Financial Applications.* Wiley.